BIRZEIT UNIVERSITY

Department of Electrical and Computer Engineering
First Semester, 2021/2022
Probability and Engineering Statistics - ENEE2307
Midterm Exam, November 22, 2021
Time Allowed: 75 Minutes.

Name: ....

ID: ....

☐ Section 1: M,W 08:30 - 09:45

☐ Section 2: T,R 10:00 - 11:15

☐ Section 3: T,R 08:30 - 09:45

☐ Section 4: T,R 12:50 - 14:05

☐ Section 5: M,W 12:50 - 14:05

Question #	Max Grade	Achieved
1	15	
2	15	
3	15	
4	15	
5	15	
Total	75	

Opening Remarks:

- This is a 75-minute exam. Calculators are allowed. Books, notes, formula sheets, and other aids are not allowed.
- You are required to show all your work and provide the necessary explanations everywhere to get full credit.

Problem#1 [8+7 Points]

A. Consider the random experiment of simultaneously tossing two fair coins. Let A denote the event that "at least one tail shows" and B denote the event that "there is a match of two coins". Compute P(A/B).

$S = \{HH, HT, TH, TT\}$

$A = \{HT, TH, TT\}, \quad B = \{TT, HH\}$

$$p(A/B) = \frac{p(A \cap B)}{p(B)} = \frac{p(\{TT\})}{p(B)} = \frac{1/4}{2/4} = 1/2$$

B. The probability that a patient recovers from COVID-19 is 0.95. What is the probability that exactly 2 of the next 3 patients recovers from COVID-19?

$p(R) = 0.95 = p(\text{success})$

X: number of patients re covered from covid-19.

$n = 3$

$$p(X = x) = \begin{cases} \binom{n}{x} p(R)^{x} \{1 - p(R)\}^{n-x} & x = 0, 1, 2, 3 \\ 0 & \text{o.w} \end{cases}$$

$$= \begin{cases} \binom{3}{x} (0.95)^{x} (0.05)^{3-x} & x = 0, 1, 2, 3 \\ 0 & \text{o.w} \end{cases}$$

$$p(X = 2) = \binom{3}{2} (0.95)^2 (0.05)^1 = \frac{3!}{(3-2)!\,2!} (0.95)^2 (0.05)$$

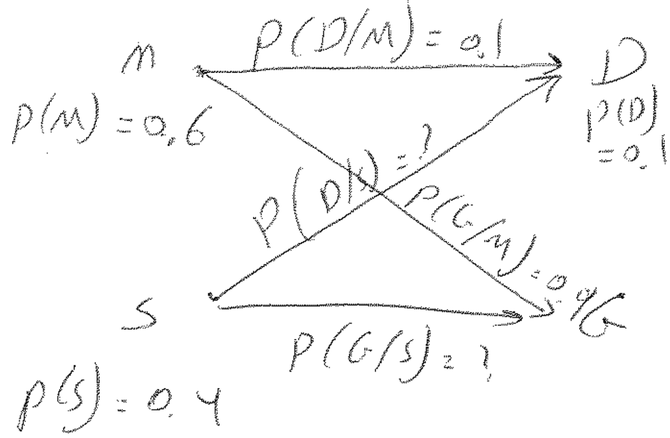
$= 0.135375$.

Problem#2 [8+7 Points]

A factory depends on two production lines for manufacturing a certain type of electronic integrated chips. The main production line (**M**) is producing 60% of the chips out of which 10% are defective (**D**) and the rest are of good quality (**G**). The secondary production line (**S**) is working only if the main production line is not working. If 10% of the total factory production is found to be defective, find the following?

a) The probability of choosing a defective chip made by the main production line.

$$p(D \cap M) = p(M)\,p(D/M)$$
$$= 0.6 * 0.1$$
$$= 0.06$$



b) If a chip is selected from the secondary production line, find the probability that it is defective.

Given $p(D) = 0.1$, $p(D/S) = ?$

$$p(D) = p(M)\,p(D/M) + p(S)\,p(D/S)$$
$$0.1 = 0.6 * 0.1 + 0.4 * p(D/S)$$
$$p(D/S) = \frac{0.1 - 0.06}{0.4} = 0.1$$

Problem#3 [5+5+5 Points]

Consider the probability density function $f_X(x)$ given below

$$f_X(x) = \begin{cases} k\sqrt{x} & 0 < x < 1 \\ 0 & otherwise \end{cases}$$

a) Find the constant k such that $f_X(x)$ is a valid pdf.

$$\int_{-\infty}^{\infty} f_X(x)\,dx = 1 \rightarrow \int_0^1 k\sqrt{x}\,dx = k\,\frac{2}{3}x^{3/2}\Big|_0^1$$

$$= \frac{2k}{3} = 1 \longrightarrow k = \frac{3}{2} = 1.5$$

b) Find the $P(0.2 \leq x \leq 0.8)$.

$$p(0.2 \leq x \leq 0.8) = \int_{0.2}^{0.8} 1.5\sqrt{x}\,dx = \frac{1.5\,x^{3/2}}{1.5}\Big|_{0.2}^{0.8}$$

$$= (0.8)^{3/2} - (0.2)^{3/2} = 0.627$$

c) Find the mean of X.

$$\mu_X = \int_{-\infty}^{\infty} x\,f_X(x)\,dx = \int_0^1 1.5\,x\sqrt{x}\,dx = \int_0^1 1.5\,x^{3/2}\,dx$$

$$= \frac{1.5\,x^{5/2}}{5/2}\Big|_0^1 = \frac{1.5}{2.5} = 0.6$$

Problem#4 [8+7 Points]

Two machines are being considered for a use in a certain production-line. The lifetime of machine A is modeled by a Gaussian random variable with a mean 20000 hours and standard deviation 4000 hours. The lifetime of machine B is also a Gaussian random variable but with a mean 24000 and a standard deviation 1000 hours.

a) What is the probability that the life time of machine A is greater than or equal to 20000 hours?

$\mu_A = 20000$, $\sigma_A = 4000$, $\mu_B = 24000$, $\sigma_B = 1000$

$$P(A \geq 20000) = 1 - P(A < 20000) = 1 - \phi\left(\frac{2000 - 2000}{4000}\right)$$

$$= 1 - \phi(0) = 1 - 0.5 = 0.5$$

b) Which machine is preferred if the target lifetime of the production-line is 22000 hours?

$$P(A \geq 22000) = 1 - P(A < 22000) = 1 - \phi\left(\frac{22000 - 20000}{4000}\right)$$

$$= 1 - \phi\left(\frac{2000}{4000}\right) = 1 - \phi(0.5) = 0.6915$$

$$P(B \geq 22000) = 1 - P(B < 22000) = 1 - \phi\left(\frac{22000 - 24000}{1000}\right)$$

$$= 1 - \phi\left(\frac{-2000}{1000}\right) = 1 - \phi(-2) = 1 - [1 - \phi(2)]$$

$$= \phi(2) = 0.9772$$

Machine B is preferred since it's probability of achieving the target lifetime is higher.

Problem#5 [6+3+3+3 Points]

The random variable X has a uniform probability density function over the interval [-2, 6]. A random variable Y is defined by the transformation $Y = (X-2)^2$.

A. Determine the mean and variance of X.

$$\mu_X = \frac{a+b}{2} = \frac{-2+6}{2} = \frac{4}{2} = 2$$

$$\sigma_X^2 = \frac{(b-a)^2}{12} = \frac{(6-(-2))^2}{12} = \frac{(6+2)^2}{12} = \frac{64}{12} = \frac{16}{3}$$

B. Find $P(X \leq 2)$

$$P(X \leq 2) = \int_{-\infty}^{2} f_X(x)\,dx = \int_{-2}^{2} \frac{1}{b-a}\,dx = \int_{-2}^{2} \frac{1}{8}\,dx$$

$$= \frac{1}{8}x\Big|_{-2}^{2} = \frac{1}{8}(2-(-2)) = \frac{4}{8} = \frac{1}{2}$$

C. Find $P(Y \le 4)$

$$P(Y \le 4) = P((X-2)^2 < 4)$$

$$= P(-\sqrt{4} < (X-2) < \sqrt{4}) = P(-2 < X-2 < 2)$$

$$= P(0 \le X \le 4) = \int_0^4 \frac{1}{8}\,dx = \frac{1}{8}x \Big|_0^4$$

$$= \frac{4-0}{8} = \frac{1}{2}.$$

D. Find $f_Y(1)$

$$f_X(x) = \begin{cases} \frac{1}{8} & -2 < x \le 6 \\ 0 & \text{o.w} \end{cases}$$

$$X = \pm\sqrt{Y} + 2$$

$$\frac{dy}{dx} = 2(X-2)$$

$$f_Y(y) = \frac{f_X(x)}{\left|\frac{dy}{dx}\right|}\Bigg|_{X=\sqrt{y}+2} + \frac{f_X(x)}{\left|\frac{dy}{dx}\right|}\Bigg|_{X=-\sqrt{y}+2}$$

$$f_Y(Y=1) = \frac{f_X(x)}{\left|\frac{dy}{dx}\right|}\Bigg|_{X=\sqrt{y}+2=\sqrt{1}+2=3} + \frac{f_X(x)}{\left|\frac{dy}{dx}\right|}\Bigg|_{X=-\sqrt{1}+2=1}$$

$$= \frac{\frac{1}{8}}{|2(3-2)|} + \frac{\frac{1}{8}}{|2(1-2)|} = \frac{\frac{1}{8}}{2} + \frac{\frac{1}{8}}{2} = \frac{1}{8}$$

Probabilities for the standard normal distribution

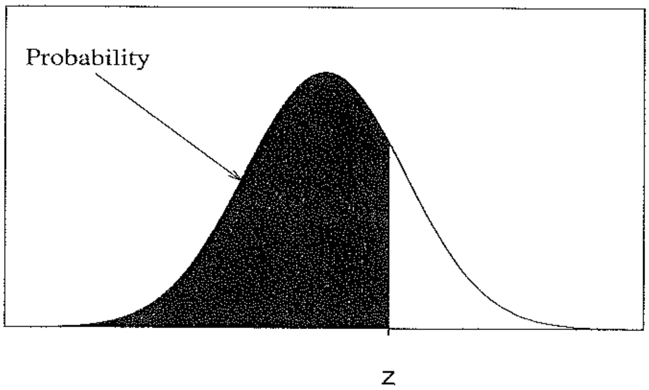


Table entry for z is the probability lying to the left of z

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998